HABILITATION THESIS REVIEWER'S REPORT

Masaryk University

Applicant	Jörgen Linus Wulff
Habilitation thesis	Integrable deformations of strings
Reviewer	Dr Daniel C. Thompson (PhD)
Reviewer's home unit, institution	Department of Physics, Swansea University, United Kingdom …

In this thesis Dr Wulff presents an impressive variety of important results concerning aspects of integrable two-dimensional quantum field theories and their target space geometrical properties. The five chapters of the thesis are lucid and very well written providing a broad introduction to the topic and highlighting the key results. The included papers are very impressive works, and are notable in that they appear in impactful Q1 journals including 2 Physics Review Letters, 6 Journal of High Energy Physics and 1 Journal of Physics A. Moreover, these manuscripts only have a small number of authors which further underscores the impressive scope of Dr Wulff's contributions. My overall summary is that work meets the requirements of a habilitation thesis, presenting a clear research vision and an important contribution to the field.

Moving on to the details of the thesis itself.

The work centres on a class of two-dimensional quantum field theories called Non-Linear Sigma Models [NLSM]. These theories are interesting in their own right as tractable but nontrivial QFTs that can exposes general features anticipated in higher dimensional QFTs. But here the real importance is that the NLSM arise in string theory as the world sheet description of strings propagating in a curved target space time geometry. In this context, the worldsheet theory should have an extra symmetry of conformal invariance. At the quantum level the demand of conformal invariance gives rises to constraints on the target space geometry that are encapsulated by a (super) gravitational theory organised in a perturbative expansion in the inverse string tension. The work of thesis bridges both the world sheet and target space properties of strings in curved backgrounds.

Dr Wulff specialises on NLSM that enjoy an enlarged infinite symmetry property know as integrability. Integrability is the special property possessed by certain physical systems in which the existence of a rich, subtle and hidden set of symmetries provides enough structure so as to render the system exactly solvable without resorting to perturbation theory.

Intuitively, we expect integrable models to be as valuable as they are rare. Only recently we have started to understand that it may be possible to explore this landscape quite systematically, starting from a known integrable system examining if it is possible to alter it in a non-trivial fashion which however retains the property of integrability. In this way we forge paths, known as integrable deformations, through the landscape of integrable systems. This thesis is centred around algebraic integrable deformations known as Yang-Baxter deformations.

A further motivation for this work, implicit in the cannon of works presented by Dr Wulff, comes in the context of the gauge-gravity holographic correspondence. When such deformations are applied to string backgrounds that are holographically dual to Quantum Field Theories they can encode supersymmetry-breaking yet integrable deformations of the holographic dictionary, thereby vastly extending the scope of holography.

In Chapter 2 provides a concise presentation of the models at hand and presents the complementary tool of non-abelian T-duality (here viewed as a canonical transformation that can be applied to map a NLSM to another NLSM). In this chapter a key result is highlighted: by including a Lie algebra cocycle term into the NLSM prior to T-dualisation one obtains a deformed T-dual [DTD] theory, which after an elaborate change of variables is shown to be exactly the Yang Baxter [YB] deformation. This is an impressive and influential result in the subject that provided a radical new construction of integrable deformations and indeed broaden the scope of non-Abelian T-duality to provide a generalisation of the familiar TsT transformations. This introduces the findings of reprint VI in which non-Abelian T-duality is considered for Green-Schwarz strings that encode supersymmetric completions of the YB deformed theories.

In Chapter 3 attention is turned to the target space description of integrable models that the author vastly clarified in his attached manuscript IX. The aim of this chapter is to explain how the YB and DTD models satisfy the leading quantum constraint of Weyl Invariance. A direct calculation in the conventional framework of (super)-gravity is unappealing in complexity and here the author makes use of a "doubled" reformulation of gravity in which metric and two-form data are unified into a single "generalised" metric. The power of this is that in this language the very involved geometry associated to YB and DTD models becomes transparent. Here the necessary conditions are obtained for Weyl Invariance and shown they are statisfied in the case of so-called uni-modular deformations (eq. 3.56). This again is an important result highlighting the quantum constraints of integrable deformed models that is further developed in reprint I. There it shown that in certain cases the uni-modular condition can in fact be relaxed leading to new examples.

In Chapter 4 the technology is pushed further by considering the string-theoretic induced corrections that complement two-derivative Einstein-Hilbert type gravity with higher derivative terms. Again this is approached in the framework of doubled geometry (more precisely in a frame-like formulation). In section 4.1 the author lucidly explains how the algebra of doubled/generalised Lorentz transformations can admit deformations that reproduce the known corrections including e.g. the spin-connection correction to the transformation of the NS two-form in the heterotic string. Armed with this in reprint II, non-abelian and the more general Poisson-Lie T-duality are examined at two-loop order. It is demonstrated how this anomalous generalised Lorentz transformation sources corrections to the duality rules that ensures PL duality extends to a solution generating symmetry at this loop order. Invoking related principles reprint III and IV study two loop conformal invariance conditions of (homogenous) Yang-Baxter deformations.

Taken together this thesis, and the canon that it presents, represent an important and influential contribution to the field of integrable models. Without question, the thesis clearly meets the requirements of a habilitation thesis in the field of Theoretical Physics and Astrophysics.

Reviewer's questions for the habilitation thesis defence

The thesis does invite some further questions that could provide interesting opportunity for discussion and possible development in the future:

1) I was taken by the suggest that the alpha' corrections to YB type deformations could resolve singularities and was wondering if this was something that Dr Wulff has further clarity on at this stage.
2) The impressive DTD procedure introduces to the PCM a cocycle term. Classically this doesn't alter theory, but I was wondering what the effects were at a quantum level; is there any view as to how e.g. the partition sum would be modified?
3) Further, given that the non Abelian T-duality is canonical transformation would it not be simpler to study and define the integrable deformations just as such a cocycle deformation? Does this remain tractable at the case of super-cosets as required for e.g. the AdS5xS5 superstring?
4) Given the wider community interest in these deformations as applied to holography, it could be insightful to understand if Dr Wulff has a perspective as to how the subject might be developing forward in this direction.

Conclusion

The habilitation thesis entitled “Integrable deformations of strings” by Jörgen Linus Wulff **fulfils the** requirements expected of a habilitation thesis in the field of Theoretical Physics and Astrophysics.

Date: 4/9/2022

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